

# Examen de L3. Probabilité. Juin 2018. Note sur 20

M53

June 9, 2018

1. Soit  $(X, Y)$  un couple de variables aléatoires de densité conjointe

$$f_{X,Y}(x, y) = 2 e^{-(x+y)} \mathbf{1}_A(x, y),$$

où

$$A = \{(x, y) \in \mathbf{R}^2; 0 \leq y \leq x\}.$$

- Déterminer les lois marginales  $f_X, f_Y$ .
  - Est-ce que  $X$  et  $Y$  sont indépendantes? Justifier la réponse.
  - Calculer  $\mathbf{E}((X - Y)^2)$ .
2. Soit  $X$  une v.a. avec densité exponentielle et  $Y$  une variable binomiale  $B(4, 1/4)$ , et supposons que  $X$  et  $Y$  soient indépendantes.
- Calculer  $\mathbf{P}(X + Y \leq t)$ .  
(*Idée*: Ecrivez  $\mathbf{P}(X + Y \leq t) = \mathbf{P}(X \leq t - Y)$  et partitionner le membre de droite sur les valeurs de  $Y$ .)
  - Montrer que  $X + Y$  a une densité et calculer cette densité.
3. On lance plusieurs fois et de manière indépendante deux dés; soit  $X$  le nombre de lancées du 1<sup>er</sup> dé nécessaires pour obtenir 1, et  $Y$  le nombre de lancées du 2<sup>ème</sup> dé nécessaires pour obtenir 5 ou 6.
- Calculer les lois de  $X$  et de  $Y$ .
  - On pose  $Z = \max(X, Y)$ . Déterminer  $F_Z(n) = \mathbf{P}(Z \leq n)$ .
  - Calculer  $\mathbf{P}(Z = n)$ .  
(*Idée*: Exprimez  $\mathbf{P}(Z = n)$  en fonction de  $F_Z(n)$  et de  $F_Z(n - 1)$  et utilisez l'item précédent).
  - En utilisant les arguments des deux items précédents, calculer la loi de  $W = \min(X, Y)$ .
  - Calculer enfin  $\mathbf{P}(X \geq Y)$ .  
(*Idée*: l'évènement  $X \geq Y$  est donné par la réunion des évènements  $X \geq y$  pris sur les valeurs  $y$  de  $Y$ .)

PRENOM:

NOM:

Filière et groupe:

LA PRESENTE PAGE DE GARDE COMPORTE LE TEXTE ET VOTRE NOM, ELLE EST A RENDRE IMPERATIVEMENT

## ANGLAIS

### COMPRÉHENSION et EXPRESSION

### ÉCRITES

L3S5 Filières SI / PC / Maths & Miashs

Durée : 1H30

**AUCUN document autorisé**

**TOUS LES TELEPHONES PORTABLES DOIVENT ETRE ETEINTS ET HORS D'ATTEINTE**

**Toute tricherie se soldera par une exclusion immédiate**

## Former Google Engineer Is Creating A Religion That Worships An AI God

1 As long as people have existed, old gods have died as new ones are conjured. And as technology and machines get ever more entwined with our lives, it is little wonder that some feel like a new religion will develop. That, at least, is the thought of former Google engineer Anthony Levandowski, who has apparently formed a new Silicon Valley-based religion, and has lofty dreams of worshipping an AI god.

5 There are scant few details about the form that this AI overlord may take. Will it be benevolent and caring, or wrathful and avenging? Will it demand sacrifices on a motherboard altar? Or require you to imbibe Juicero while performing hot yoga? Who knows, for the time being we'll just have to wait until further instruction from our future god, and hope we do nothing to anger it in the meantime.

10 The germinating religious sect was uncovered by Wired, who found that in 2015 Levandowski founded a religious organization with the aim to "develop and promote the realization of a Godhead based on Artificial Intelligence," which has been called rather ominously *Way of the Future*. They also found documents that showed how Levandowski has positioned himself as the CEO and President of Way of the Future. Perhaps he's trying to curry favor with the new supreme leader.

15 The community that has arisen in Silicon Valley has already been described by many as akin to a cult worshiping technology, so maybe there is a place for this new order. Coders, engineers, and developers talk about the moment that machines surpass humans in all aspects as the "singularity", when AI becomes so intelligent and capable that we as a biological species won't even be able to comprehend what it knows.

20 As technology has evolved, humans have always adapted and changed their religion and deities to fit with the changing times. From the advent of agriculture killing the spirits worshipped by hunter-gatherers, or perhaps even the god of fire and volcanoes being co-opted by steelworkers, this is a common occurrence in communities throughout history. So perhaps the development of a deity based on computing, robotics, and cyberspace is not that surprising in the long run.

25 Either way, it might be a while before Levandowski can devote his entire attention to the new Godhead, as he is currently mired in a billion dollar lawsuit between two of the biggest plays in Silicon Valley, as two of his former employees face off against each other. Google is claiming that Levandowski stole trade secrets on self driving cars in a bid to replicate the technology for his then new employer Uber, and is seeking a pretty hefty \$1.9 billion in damages.

Maybe this is the natural progression of things. It's been argued before that we worship technology already, perhaps this is just putting a face to it all.

**PART ONE : READING COMPREHENSION**

**1. Identify what / who the underlined words in the text at page 2 refer to. (5 points)**

1. ones (line 1) refers to .....
2. some (line 2) refers to .....
3. it (line 5) refers to .....
4. they (line 11) refers to .....
5. he (line 13) refers to .....
6. it (line 17) refers to .....
7. he (line 24) refers to .....
8. each other (line 25) refers to .....
9. we (line 28) refers to .....
10. it (line 29) refers to .....

**2. Translate the following sentences into English. The underlined sections are to be found in the text, mostly in the first two paragraphs. (10 points)**

1. La technologie et les téléphones portables sont de plus en plus enchevêtrés dans nos vies. .....
2. Il a des idées ambitieuses pour vénérer l'Intelligence artificielle. .....
3. Il se peut que ce nouveau dieu exige des sacrifices sur l'autel de la carte-mère. .....
4. Il nous faudra patienter jusqu'à ce que de nouvelles instructions nous soient données. .....
5. Pendant ce temps-là, on espère que l'on ne fera rien à notre professeur qui le mette en colère. .....

**3. Right or Wrong? Justify precisely from the text. (10 points)**

**a) As long as there will be people, there will be gods, too.**

- right
- wrong

Justify : -----

**b) Anthony Levandowski is still working in the Silicon Valley.**

- right
- wrong

Justify : -----

**c) A god may experience as many emotions as a human.**

- right
- wrong

Justify : -----

**d) We know very little about the shape of Artificial Intelligence.**

- right
- wrong

Justify : -----

**e) The religious group founded by A. Levandowski just began to spread in 2015.**

- right
- wrong

Justify : -----

**4. In §3, pick up the adverb showing that the religious use of Artificial Intelligence sounds rather threatening : ----- then translate it into French : -----**  
(2 points)

**5. What do we know about Levandowski? Tick everything that is correct.**

- He invented a new religion based in California
- He was used to having a position as engineer for Google
- He loves himself so much that he sees himself worshipped by many people
- He has ambition and shows a true sense of leadership
- He is going to work for a new tech company called *Way of the Future*
- He thinks Artificial Intelligence is the basis of people's future life
- He certainly wants to gain President Trump's favour

6. Read §4, then explain in FRENCH why a new "religion" is likely to appear. (2 points) -----

-----  
-----  
-----

7. (§4) The choice of gods has long been depending on human activities. (1,5)

- right
- wrong

Justify : -----

8. (§4) Hunter-gatherers were more likely to worship the god of earth. (1 point)

- right
- wrong

(NO JUSTIFICATION REQUIRED)

9. Explain briefly (*in French if you want*) why steelworkers would choose the god of fire rather than the god of water?! (2 points) -----

-----  
-----

10. The idea of high technology as a god is now almost thought of as normal. (1 point)

- right
- wrong

Justify : -----

11. Who is likely to pay \$1.9 billion in damages and why? Explain in FRENCH. (2 points) -----

-----  
-----  
-----  
-----

12. The writer of this article sounds fatalistic rather than optimistic. (1,5)

- right
- wrong

Justify : -----

**PART TWO : WRITING ENGLISH (20 points)**

**1) Write FIVE different questions you would ask Anthony Levandowski after reading the text. (5 points)**

1. -----
2. -----
3. -----
4. -----
5. -----

**2) Are you keen on high technology? Which device is your favourite? How do you explain that some people regard it as dangerous? (+ / 70 words, 8 points)**

-----  
-----  
-----  
-----  
-----  
-----  
-----  
-----  
-----  
-----

**3) Which topic did you prefer studying this semester? Which did you not like? Explain why. (+ / 70 words, 7 points)**

-----  
-----  
-----  
-----  
-----  
-----  
-----  
-----  
-----  
-----

L3 MATHS

M62

Examen du 2 juillet 2018

*Les deux parties sont à traiter sur des copies séparées*

PARTIE I

**Exercice 1** (3pts).

(i) Soit  $f$  la fonction de  $\mathbb{R}^2$  dans  $\mathbb{R}$  définie par

$$f(x, y) := \begin{cases} \frac{x^3 y}{x^4 + y^2} & \text{si } (x, y) \neq (0, 0) \\ 0 & \text{si } (x, y) = (0, 0) \end{cases}$$

$f$  est-elle continue, différentiable, de classe  $C^1$  sur  $\mathbb{R}^2$  ?

(ii) Soit  $g$  la fonction de  $\mathbb{R}^2$  dans  $\mathbb{R}$  définie par

$$g(x, y) := \begin{cases} \frac{xy^3}{x^4 + y^2} & \text{si } (x, y) \neq (0, 0) \\ 0 & \text{si } (x, y) = (0, 0) \end{cases}$$

$g$  est-elle continue, différentiable, de classe  $C^1$  sur  $\mathbb{R}^2$  ?

**Exercice 2** (3pts).

Soit  $f$  la fonction de  $\mathbb{R}^2$  dans  $\mathbb{R}$  définie par

$$f(x, y) = x^2 + 2y^2 + 2xy - 18x - 24y + 120$$

(i) Etudier les éventuels extrema locaux de  $f$  sur  $\mathbb{R}^2$ .

(ii) Etudier les éventuels extrema globaux de  $f$  sur  $\mathbb{R}^2$ .

**Exercice 3** (4pts).

Soit  $f$  la fonction de  $\mathbb{R}^2$  dans  $\mathbb{R}$  définie par

$$f(x, y) = x^3 + y^3 - 3xy - 1$$

(i) Montrer que la condition  $f(x, y) = 0$  définit au voisinage de  $(0, 1)$  une fonction implicite  $\phi$  qui à  $x$  associe  $y$ .

(ii) Déterminer le développement limité à l'ordre 3 de  $\phi$  au voisinage de 0.



## PARTIE II

### Exercice 1 (4pts).

Soit le système d'équations différentielles

$$\begin{cases} y'(t) - y(t) - 2x(t) = 0 \\ x'(t) - x(t) - 8y(t) = 0 \end{cases}$$

Déterminer l'ensemble des solutions de ce système.

### Exercice 2 (2pts).

Soit  $q : \mathbb{R} \rightarrow \mathbb{R}$  une application donnée que l'on suppose continue sur  $\mathbb{R}$ . Justifier que le système d'inconnue la fonction  $x(\cdot)$

$$\begin{cases} x''(t) - q(t)x(t) = 0 \\ x(0) = 1, \quad x'(0) = 1 \end{cases}$$

admet une unique solution définie sur tout  $\mathbb{R}$  (que l'on ne calculera pas).

### Exercice 3 (4pts).

1. On considère le problème de Cauchy non linéaire suivant

$$(E) \quad \begin{cases} y'(x) = e^{-xy(x)} \\ y(0) = 1 \end{cases}$$

Montrer l'existence et l'unicité d'une solution maximale  $(I, y_0)$  de  $(E)$ .

2. Montrer que cette solution  $(I, y_0)$  de  $(E)$  est impaire et strictement croissante.
3. Écrire l'équation intégrale associée à  $(E)$ . En utilisant la question (2) et le fait que  $y_0(0) = 1$ , en déduire que  $I = \mathbb{R}$ .