Examen, durée : 1h30

- Compléter les mots dont on vous donne la première lettre ainsi qu'un synonyme entre parenthèses. (4 pts)

1. We are currently flying at a h................ of 15,000 metres. (altitude)
2. The car was only sl................ damaged after the accident. (a little)
3. He paid regular am.................. of money to a charity. (sums, quantities)
4. They finally re.................. the coast after five weeks sailing. (attained)
5. The river froze to a de................ of over a metre. (distance down)
6. She picked up a th................... volume and began to read out loud. (thick)
7. As it approaches the sea, the river begins to wi................ . (get broader)
8. Dress sizes ra.................. from petite to extra large. (vary)
9. Yesterday a satellite we.................. 15 tonnes was successfully placed in orbit. (having a mass of)
10. The ar.................. of New York to the south of Houston Street is known as Soho. (zone, district)
11. 3, 5, 7 are all o.................. numbers. (even)
12. S.................. do we receive any apology when mistakes are made. (rarely)
13. She ren.................. her efforts to escape. (repeated)
14. The pa.................. of family life has been changing over recent years. (model)
15. Most of these people are paid mo................ . (every month)
16. The books were ra.................. arranged on the shelves. (done or chosen by chance rather than according to a plan)
17. He walked briskly along the path sw................ his rolled-up umbrella. (moving from one side to the other)
18. The doctor could feel no pulse be.................. (making a regular movement)
19. He phones me every o.................. day. (second)
20. She h.................. e.................. goes to London. (very rarely)
21. The report claimed that the r.................. of alcoholic journalists was 1 : 10. (proportion)
22. The a.................. temperature during the month of June in London is approximately 16°C. (mean)
23. The poor are st .................. getting poorer. (gradually, constantly)
24. They hope to reduce the air pollution r.................. by using more electrical transport. (level)
• **QCM « grammaire et structures » Choisir la bonne réponse (4pts)**

1. These goods .................. at least three weeks ago.
   a) must deliver  
   b) must be delivered  
   c) must have been delivered  
   d) ought to be delivered

2. The workers .................. on strike .................. three months.
   a) have been / for  
   b) have been / since  
   c) were / ago  
   d) are / since

3. The government would fall if those ministers .................. their support.
   a) withdraw  
   b) withdrew  
   c) had withdrawn  
   d) have withdrawn

4. How long ago .................. their London flat?
   a) have they been selling  
   b) had they sold  
   c) have they sold  
   d) did they sell

5. I think .................. him yesterday.
   a) I’ve met  
   b) I’d met  
   c) I was meeting  
   d) I met

6. This is the first real failure we have had ever since ..................  
   a) we started  
   b) we have started  
   c) our start  
   d) we have been started

7. Their window .................. yet.
   a) has not repaired  
   b) was not repaired  
   c) has not been repairing  
   d) has not been repaired

   a) had  
   b) has  
   c) was  
   d) did

9. There is evidence that the number of such specialists .................. over the last ten years.
   a) is increasing  
   b) increases  
   c) has increased  
   d) should be increasing

10. Peter and I .................. a drink when the fire broke out in the pub.
    a) had  
    b) were having  
    c) have had  
    d) used to have
11. He ................................ unemployed for six months when he received their offer.
   a) was                   b) would be
   c) has been             d) had been

12. "Have you visited the City of London?" -- "Yes, as a matter of fact, I ................................ there last week."
   a) have been           b) went
   c) have gone           d) have been going

13. ................................ borrow your pen for a second, just to sign this report?
   a) Am I able to       b) Must I
   c) Would I             d) Could I

14. "Why doesn't he answer the phone?" -- "He ................................ be away, I suppose."
   a) ought to             b) should
   c) must                   d) has to

15. She was surprised to find that he ................................ remember her name at once.
   a) could not           b) cannot
   c) is not able to       d) should not be able to

16. What is he complaining about? He ................................ have been more careful.
   a) must              b) should        c) may           d) would

17. One shouldn't lose ................................ sense of proportion.
   a) your                  b) the
   c) one's                 d) their

18. It pleases Granny to see the children enjoying .................................
   a) themselves         b) herself
   c) oneself              d) ourselves

19. I don't have a bike, but Mike does. Maybe I can use .................................
   a) of his              b) his
   c) of him               d) his one

20. Last Sunday we went and called upon a relative .................................
   a) of us              b) ours
   c) of our               d) of ours

21. We had a bottle of milk with us. I poured ................................ a full glass.
   a) myself          b) to myself
   c) me                d) to me

22. "It's very cold outside."
   "Yes indeed; we'd better ................................ the window."
   a) to not open       b) don't open
   c) not to open        d) not open

23. Her pain was not as bad as it ................................ been.
   a) might have       b) need have
   c) must have        d) can have
24. She ................ wory, I'm confident she will succeed.
   a) need not to   b) need not   c) does not need   d) needs not to

25. I was late. The train ................ the station by the time I got there.
   a) left         b) had left
   c) had been leaving   d) would leave

26. He swam much faster than ............... did.
   a) one         b) she
   c) her   d) them

27. Does he usually talk to ................ ?
   a) herself      b) each other
   c) oneself      d) himself

28. He couldn't answer the question because he ................ for the last ten minutes.
   a) was dreaming      b) was daydreaming
   c) had been daydreaming   d) had daydreamed

• Complete the news report. Put each verb into the correct form. (4pts)

An eight-year-old boy ........ has disappeared ........ disappear

Mark Davidson ....................... yesterday from the park near his home in 1 not return

Cotley Road, Dulverstone. The Davidsons ....................... 2 live

five minutes' walk away from the park.

Mark ...................... to the park at four o'clock and 3 go

4 ...................... football with his friends. After they 4 play

5 ...................... for about an hour, they 5 play

6 ...................... down for a rest. Mark 6 sit

7 ...................... the park at quarter past five. He 7 leave

8 ...................... alone. A man who 8 be

9 ...................... his dog 10 ....................... 9 walk 10 see

him go out through the gate. No one 11 ....................... 11 see

Mark since then. Police 12 ....................... local residents,

and so far they 13 ....................... to about two hundred 13 speak

people. They 14 ....................... to question a man in a green sweater who 14 want
15. ......................... on the grass near the park exit at five o'clock.       15 lie

At the moment police and other helpers 16. .............................. the nearby
fields and woods in the hope of finding the missing boy.

• Questions about the texts studied in class:

1. How important was Einstein’s contribution to science? Why did he become involved in politics? What were his ethics and do you approve of them or not? (4pts)

2. What do you think of Stephen Hawking both as a scientist and as a person? Why did you choose to study science? Explain. (4pts)
Université du Sud Toulon-Var  
UFR des Sciences et Techniques

L3 mathématiques

Anglais E51  seconde session le 22 juin 2011

durée : 1h30

- Compléter les mots dont on vous donne la première lettre ainsi qu’un synonyme entre parenthèses. (3pts)

1. The plane usually reaches its maximum h................. of 36,000 feet in about ten minutes. (altitude)
2. The waiting time was s................. less than four hours. (rarely)
3. Unemployment has decreased st................. (regularly)
4. It was 25 metres long and 30 tons in w................. . (mass)
5. His description had been reasonably ac................. . (correct).
6. He started to swim from the s................. end of the pool. (not deep)
7. They stock a wide r................. of electrical goods. (variety)
8. I was horrified by the a................. of work I had to do. (quantity)
9. A heat w................. is a period of time during which the weather is much hotter than usual. (sudden increase)
10. The region has suffered an unprecedented se................. of natural disasters. (sequence, succession)
11. Their studies revealed a consistent pa................. in the history of the country’s economy. (model)

12. This chapter includes a brief su................. of the most commonly used drugs. (study)
13. I have at this stage no f................. comment on them. (additional)
14. Their local committees are usually held every o................. month. (every two months)
15. In 1959 the a................. age of teachers was thirty-nine years. (mean)
• must have or can't have (1.5pts)

Put in **must have or can't have** and the past participle of the verb on the right:

Mike: I can't wind the film on in this camera. (faire avancer la pellicule)

Robert: You **must have finished** the film, then.

Mike: I (1) ........................................ a whole film. I've only taken four photos. I use

Robert: Well, you(2).............................. the whole film in correctly, then. 2 put

You (3) ........................................... it wrong. You'll have to take it out. 3 do

• must, mustn't or needn't?

Put in **must, mustn't or needn't.** (2.5pts)

Mother: You .... **needn't**... take an umbrella. It isn't going to rain.

Son: Well, I don't know. It might do.

Mother: Well, look after it, please. You **mustn't** lose it.

1. Mervyn: Come on. We.............................hurry. We.........................be late.

   Isabel: It's only ten past. We.............................hurry. We've lots of time.

2. Pupil: Jason and I are going for a walk

   Teacher: No, you .........................go off on your own. I want you all together.

   We..........................keep together.

3. Sandra: I'll put these glasses in the dishwasher.

   Natalie: No, you.........................put them in there. They might break. In fact, We..........................wash them at all. We didn't use them.

4. Secretary: I .........................forget to type this letter.
Boss: It .................go in the post today because it's urgent. But the report isn't so important. You..........................type the report today.

- **READING COMPREHENSION:** Choose the best answer. Only one answer is correct. (4pts)

Psychologists have found that privately made confidential resolutions are rarely followed, whereas a public commitment to achieve some goal, such as losing weight or giving up smoking, is likely to be much more effective. This is because the approval of others for doing something desirable is valued. In contrast, disapproval for failure can lead to feelings of shame.

Advertising agencies have designed studies bearing out the truth of this observation. In this research, a group of strangers was bombarded with information about the qualities of a particular product. They were then asked to either announce out loud or write out privately whether they intended to buy the product. It was later discovered that those who publicly declared their intention to buy were considerably more likely to do so than those who affirmed their intentions in private.

In another study, an experimenter claiming to represent a local utility company interviewed house owners telling them he was investigating ways in which energy consumption could be reduced. Half the subjects, randomly selected, were told that if they agreed to conserve energy their names would be mentioned in an article published in the local newspaper; the remaining half were told their names would not be used. All those interviewed agreed to cooperate and signed a form either giving consent for their names to be used or stating that their names would not be used. Later in the year, the amount of gas consumed in each house was recorded. The owners who had agreed to their names being published had used significantly less gas than those who remained anonymous.

1. What is the main topic of the passage?
   (A) The commitment to conserve energy
   (B) The effectiveness of public commitment
   (C) The results of studies done on advertising agencies
   (D) The observations of the effects of advertising

2. It can be inferred that all of the following help motivate a person to achieve a goal EXCEPT
   (A) a desire for approval
   (B) a fear of disapproval
   (C) a fear of failure
   (D) a sense of noncommitment
3. The word "shame" in line 5 is closest in meaning to
   (A) anger
   (B) disgrace
   (C) humility
   (D) inadequacy

4. Why were advertising agencies probably interested in conducting their study?
   (A) They wanted to introduce their people to more products
   (B) They wanted to demonstrate the quality of their products
   (C) They wanted to know if people intended to purchase their products
   (D) They wanted to find the best way to get people to buy their products

5. The word "bombarded" in line 7 is closest in meaning to
   (A) bombed
   (B) attacked
   (C) saturated
   (D) hampered

6. According to the passage, the anonymous subjects in the energy-consumption experiment
   (A) didn't cooperate
   (B) didn't sign a form
   (C) consented to have their names published
   (D) didn't use significantly less gas

7. How did the experimenters find out how much gas the subjects used?
   (A) The amount was recorded
   (B) The amount was stated in the contract
   (C) The people published the amount
   (D) The people were given a limited amount

8. The passage supports which of the following conclusions?
   (A) Commitments made in private are most likely to fail.
   (B) Disapproval for failure makes people less willing to make public commitments.
   (C) Intentions announced out loud are more effective than those published in newspapers.
   (D) Well-informed people are more likely to publicly declare their intentions.

• ESSAY (9pts) [180 words minimum]

Write about an environmental problem that you feel concerned about. Give examples of things every citizen can do every day to preserve the environment.
EXAMEN DE M62 Mai 2011: les notes de cours sont autorisées. La note est sur 20; il y a 25 points pour les questions suivantes.

1. Soit \((X, \mathcal{A}, \mu)\) un espace mesuré avec \(p \in [1, \infty)\); soit ensuite \(f_n, n \in \mathbb{N}\), une suite dans \(L^p(X, \mathcal{A}, \mu)\) et \(f \in L^p(X, \mathcal{A}, \mu)\) telle que (\(||\cdot||_p\) denote la norme \(L^p_\mu\)):
\[
\lim_{n \to \infty} ||f_n - f||_p = 0
\]
Soit après \(g_n\) une suite de fonctions \(\mathcal{A}\)-mesurables et telle que:
(i) \(|g_n(x)| \leq M, \forall x \in X\) and \(\forall n \geq 1\);
(ii) la limite suivante existe \(\mu\)-p.p. et elle définie la fonction mesurable \(g\): \(\lim_{n \to \infty} g_n = g\).
Montrer que
\[
\lim_{n \to \infty} ||g_nf_n - g\,f||_p = 0
\]
(Suggestion: ajouter et enlever à \(g_nf_n - g\,f\) la quantité \(g_nf\) et estimer séparément les couples de termes ainsi obtenus.) (5 pts).

2. NB: dans cet exercice les sigma-algèbres sont celle de Borel et les mesures celles de Lebesgue; donc les intégrales pourront éventuellement se ramener (pour le calcul), à des intégrales de Riemann. Les deux parties sont indépendantes.

- Considérons la fonction \(f : [0,1] \to \mathbb{R}\) mesurable. Déterminer pour quelle classe de fonctions mesurables \(f\) la limite suivante est finie:
\[
\lim_{n \to \infty} \int_{[0,1]} \frac{dt}{\sqrt{f(t)^2 + \frac{1}{n}}}
\]
(Suggestion: Étudier la suite: \(f_n = \frac{1}{\sqrt{f(t)^2 + \frac{1}{n}}} \). (3 pts.)

- Calculer la limite suivante quand \(n \to \infty\):
\[
\int_{0}^{\infty} \frac{\sin(\pi x)}{1 + x^n} \, dx
\]
(3 pts.)

3. Soient \(f_n\) et \(f\) des fonctions mesurables sur l'espace mesuré \((X, \mathcal{A}, \mu)\), avec \(f_n\) et \(f\) dans \(L^p_\mu, \forall n \in \mathbb{N}, p \in [1, \infty)\). On fait les hypothèses que:
(i) \(f(x) = \lim_{n \to \infty} f_n(x)\), pour \(x - \mu\) p.p.;
(ii) \(\lim_{n \to \infty} ||f_n||_p = ||f||_p\).
• A partir de la convexité de la fonction \( t \mapsto |t|^p \), montrer que

\[
\left| \frac{f - f_n}{2} \right|^p \leq \frac{|f|^p + |f_n|^p}{2}
\]

et que donc la fonction \( \phi_n := 2^{p-1}(|f|^p + |f_n|^p) - |f - f_n|^p \geq 0 \), \( \mu \)-p.p. (2 pts.)

• En utilisant Fatou appliqué à la suite \( \phi_n \), montrer que

\[
\limsup_{n \to \infty} \int_X |f - f_n|^p d\mu = 0
\]

(2 pts.)

• Montrer pour terminer que

\[
\lim_{n \to \infty} \|f - f_n\|_p = 0
\]

(2 pts.)

4. • Considérons la mesure de Lebesgue produit, simplement indiquée ici par \( dx \, dy \) sur le carré ouvert \((1, \infty) \times (1, \infty)\). Calculer à l'aide du théorème de Fubini que l'intégrale:

\[
\int \int_{(1, \infty) \times (1, \infty)} (x + y)^{-3} dx \, dy
\]

(4 pts.)

• Soit \((X, \mathcal{A}, \mu)\) un espace mesuré avec \( \mu \) une mesure \( \sigma \)-finie. Prenons par la suite l'espace mesuré \((\mathbb{R}_+, \mathcal{B}, m)\) où \( m \) est la mesure de Lebesgue sur la \( \sigma \)-algèbre de Borel \( \mathcal{B} \) sur la demi-droite positive \( \mathbb{R}_+ = [0, +\infty) \). Considérons ensuite \( f \in L^p_m \) avec \( p \in [1, \infty) \) et enfin l'espace produit \((X \times \mathbb{R}_+, \mathcal{A} \times \mathcal{B}, \mu \times m)\).

Montrez à l'aide du théorème de Fubini que

\[
\int \int_{X \times \mathbb{R}_+} F(x, u) d(\mu \times m) = \|f\|_p^p = p \int_0^\infty u^{p-1} \mu(x; f(x) > u) du
\]

où \( F(x, u) = pu^{p-1}1_{(x, f(x)) > u} \), et \( 1_A \) dénote la fonction caractéristique de \( A \). (4 pts.)
Examen de MESURE ET INTEGRATION-M62

Les 4 exercices sont indépendants. Durée: 2H.

I- Soit \( g \) une fonction mesurable de \( [0, +\infty) \) à valeurs dans \([0, 1]\) et pour tout \( n \in \mathbb{N}^* \) posons

\[
f_n = \frac{1}{n} [1 - (1 - g)^n], \quad I_n := \int_0^{+\infty} f_n dm
\]

où \( m \) désigne la mesure de Lebesgue sur \( \mathbb{R} \).

1) Montrer que \( 0 \leq f_n \leq f_1 = g \). En déduire que \( I_n < +\infty \) pour tout \( n \in \mathbb{N}^* \) si et seulement si \( \int_0^{+\infty} g \, dx < +\infty \). Calculer dans ce cas et en justifiant la valeur de \( \lim_{n \to \infty} I_n \).

2) Établir que pour tout \( x \), la suite \((n f_n(x))\) est monotone croissante. En déduire que

\[
\lim_{n \to \infty} n I_n = m(\{g > 0\}) .
\]

II- Soit \( \mu \) une mesure de Radon bornée sur \( \mathbb{R}^N \). On note \( B(x, r) \) la boule ouverte centrée en \( x \) de rayon \( r > 0 \), \( D \) l'ensemble des atomes de \( \mu \), i.e.:

\[
D := \{x \in \mathbb{R}^N : \mu(\{x\}) > 0\} .
\]

1) Montrer que pour tout \( x \notin D \), on a: \( \lim_{r \to 0} \mu(B(x, r)) = 0 \).

2) Montrer que tout \( k > 0 \), l'ensemble \( D_k := \{x \in \mathbb{R}^N : \mu(\{x\}) > \frac{1}{k}\} \) est vide ou bien contient un nombre fini d'éléments. En déduire que \( D \) est au plus dénombrable.

III- Soit \( \sharp \) la mesure de comptage et \( m \) la mesure de Lebesgue sur \( \mathbb{R} \). En appliquant la formule de Fubini à la mesure produit \( \mu = \sharp \otimes m \), calculer de deux façons différentes \( \mu(A) \) où \( A := \{ (x, x) : x \in [0, 1] \} \). Conclusion ?

IV- On considère l'intégrale dépendant du paramètre \( \alpha \in \mathbb{R} \):

\[
I(\alpha) := \int_{\mathbb{R}^3} \frac{dx dy dz}{(1 + x^2 + y^2 + z^2)^{\alpha}} .
\]

1) Transformer cette intégrale en faisant le changement de variables \( x = r \cos \theta \cos \varphi \), \( y = r \sin \theta \cos \varphi \), \( z = r \sin \varphi \), et en précisant bien la valeur du déterminant Jacobien associé. Trouver une condition nécessaire et suffisante sur \( \alpha \) pour que \( I(\alpha) < +\infty \).

2) Calculer la limite de \( I(\alpha) \) quand \( \alpha \to +\infty \).

FIN
Exercice
Dans le repère orthonormé direct \((O, \vec{e}_1, \vec{e}_2, \vec{e}_3)\), on considère \(\mathcal{D}\) un cylindre droit de section circulaire défini par \(\{(R, \theta, z) \in [0, R] \times [0, 2\pi] \times [0, \ell]\}\)
Il est constitué d'un matériau homogène élastique linéaire dont on notera \(\lambda\) et \(\mu\) les coefficients de Lamé, \(E\) le module d'Young et \(\nu\) le coefficient de Poisson.

Ce domaine est en équilibre lorsqu'il est soumis à une densité d'efforts volumiques \(\rho \omega^2 r e_r\), qu'il peut subir un déplacement \(u\) et qu'il est soumis à un état de contraintes \(\sigma\) tels que :
- La surface latérale est libre de contraintes
- Les faces \(x_3 = 0\) et \(x_3 = \ell\) sont soumises à une densité d'efforts parallèle à \(e_3\) et le déplacement des points de ces surfaces se font exclusivement dans le plan \((e_1, e_2)\)

1. Ecrire les équations du problème
2. On cherche la solution en déplacements sous la forme \(u = f(r) e_r\). Établir l'équation différentielle satisfaite par \(f(r)\) et résoudre complètement le problème.

Problème
I. Déformations planes
On étudie un problème d'élasticité sous l'hypothèse des petites perturbations. Dans la base orthnoromée directe \((\vec{e}_1, \vec{e}_2, \vec{e}_3)\), on suppose que le déplacement dans la structure étudiée est plan et invariant par translation dans la direction \(\vec{e}_3\). Le déplacement est alors du type \(\bar{u} = (u_1, u_2, 0)\) ou \(u_1\) et \(u_2\) sont des fonctions de \((x_1, x_2)\)

1. Écrire la forme générale du tenseur des déformations dans ce cas.
2. Rappeler la loi de comportement inverse qui permet de calculer le tenseur des déformations \(\varepsilon\) en fonction du tenseur des contraintes \(\sigma\), du module d'Young \(E\) et du coefficient de Poisson \(\nu\) et montrer que \(\sigma_{33}\) est lié aux autres composantes du tenseur des contraintes.
3. Écrire les équations d'équilibre. Dans toute la suite on supposera l'absence de forces volumiques. Montrer alors qu'il existe une fonction scalaire \(F(x_1, x_2)\) telle que \(\sigma\) s'exprime en fonction des dérivées secondes de cette fonction.
   En déduire l'expression du tenseur \(\varepsilon\) en fonction de \(F, E\) et \(\nu\)
4. Quelle équation aux dérivées partielles doit satisfaire la fonction \(F\) pour que les conditions de Beltrami soit vérifiées
II. Exemple d’application

On considère un massif élastique semi infini qui occupe le domaine défini par :

\[ \Omega = \{ x_1 \in [-a, a], \ x_3 \in [-b, b], \ et \ x_2 \geq 0 \}. \]

En l’absence d’efforts volumiques, il est en équilibre dans les conditions suivantes :

- sur \( |x_1| = a \) : \( F_1 = F_3 = 0 \) et \( u_2 = 0 \)
- sur \( |x_3| = b \) : \( F_1 = F_2 = 0 \) et \( u_3 = 0 \)
- le long du plan \( x_2 = 0 \), il est soumis à des pressions normales de la forme \( P \cos \omega x_1 \vec{e}_2 \)
  et les contraintes sont nulles à l’infini.

1. Écrire les équations du problème

On s’intéresse ici aux solutions de la forme \( F = f(x_2) \cos \omega x_1 \) avec \( \omega = \frac{\pi}{2a} \)

2. Trouver l’équation différentielle à laquelle \( f \) satisfont pour que les conditions de Beltrami soient vérifiées.

3. Résoudre complètement l’équation vérifiée par \( f(x_2) \) pour que \( F \) conduise à un champ de contraintes admissibles. (Rem : On pourra chercher la fonction \( f(x_2) \) sous la forme \( f(x_2) = P_1(x_2) e^{\omega_1 x_2} + P_2(x_2) e^{\omega_2 x_2} \) où \( P_1 \) et \( P_2 \) sont des polynômes de \( x_2 \) à déterminer et \( \omega_1 \)
  et \( \omega_2 \) sont des constantes à déterminer aussi).

4. Calculer le déplacement associé au tenseur des contraintes ainsi obtenu. A-t-on déterminé
  la solution ?
Examen de Mathématiques, Licence 3ème année
Calcul Différentiel

Préambule : aucun document ni calculatrice n’est autorisé pour cette épreuve. Toutes les réponses doivent être soigneusement justifiées.

I) Soit \( f : \mathbb{R}^2 \to \mathbb{R} \) définie par :
\[
f(x_1, x_2) := \frac{x_1 x_2(x_1^2 - x_2^2)}{x_1^2 + x_2^2} \quad \text{si} \ (x_1, x_2) \neq (0, 0) \quad \text{et} \quad f(0, 0) = 0.
\]

1) Montrer que la fonction \( f \) est continue.
2) Montrer que les dérivées partielles de \( f \) existent et sont continues dans \( \mathbb{R}^2 \setminus \{(0, 0)\} \).
3) Calculer les limites des dérivées partielles en \((0, 0)\).
4) La fonction \( f \) est-elle différentiable dans \( \mathbb{R}^2 \setminus \{(0, 0)\} \) ?
5) La fonction \( f \) est-elle différentiable à l’origine ? Dans l’affirmative, expliciter \( Df(0, 0). (h_1, h_2) \) pour tout \((h_1, h_2) \in \mathbb{R}^2\).
6) Montrer que les dérivées partielles secondes existent dans \( \mathbb{R}^2 \setminus \{(0, 0)\} \).
7) Montrer que
\[
\frac{\partial^2 f}{\partial x_1 \partial x_2}(0, 0) \quad \text{et} \quad \frac{\partial^2 f}{\partial x_2 \partial x_1}(0, 0)
\]
existent et sont différentes. Que pouvez-vous en conclure ?

II) Soient \( \Omega := \mathbb{R}^2 \setminus \{(0, 0)\} \) et \( F : \Omega \to \mathbb{R}^2 \) définie par :
\[
F(x_1, x_2) = \left( \frac{x_1}{x_1^2 + x_2^2}, \frac{-x_2}{x_1^2 + x_2^2} \right).
\]

1) Montrer que \( F \) est de classe \( C^1 \) dans \( \Omega \) et expliciter
\[
DF(x_1, x_2). (h_1, h_2)
\]

pour tout \((x_1, x_2) \in \Omega \) et tout \((h_1, h_2) \in \mathbb{R}^2\).
2) Déterminer l’application \( G : \Omega \to \mathbb{R}^2 \) satisfaisant à :
\[
\forall (x_1, x_2) \in \Omega \quad \forall (y_1, y_2) \in \Omega \quad (y_1, y_2) = F(x_1, x_2) \iff G(y_1, y_2) = (x_1, x_2).
\]
3) Montrer que \( F \) est un difféomorphisme de \( \Omega \) sur \( \Omega \) puis calculer \( DG(y_1, y_2). (k_1, k_2) \)
pour tout \((y_1, y_2) \in \Omega \) et tout \((k_1, k_2) \in \mathbb{R}^2\).