Examen d'Algèbre  
Première session  
Licence de Mathématiques, troisième année

**Avertissement.** Toutes les réponses doivent être justifiées. Aucun document autorisé à l'exception d'un résumé de vos notes personnelles sur 2 feuilles maximum, recto verso, au format A4. Attention: ce résumé doit être manuscrit et non photocopié.

**Problème 1.** Sur $G := \mathbb{R}^* \times \mathbb{R}$ on définit la loi de composition interne notée multiplicativement suivante:

$$(a, b)(c, d) := (ac, bc + d).$$

(a) Vérifier que c'est bien une LCI.  
(b) Vérifier que cette LCI fait de $G$ un groupe. Est-il abélien?  
(c) Soit $H := \{1\} \times \mathbb{R}$. Vérifier que $H$ est un sous-groupe de $G$.  
(d) Vérifier que $H$ est distingué.  
(e) Caractériser les éléments du quotient $G/H$.  
(f) Soit $A$ le sous-ensemble de $\mathbb{R}^2$ défini par

$$f \in A \iff \exists (a, b) \in G, \quad f(x) = ax + b.$$  

Montrer que $A$ muni de la LCI: $(f, g) \mapsto g \circ f$ où $\circ$ désigne la composition des des applications, est un groupe.  
(g) Montrer que $A$ est isomorphe à $G$.  
(h) Quelle est l'image de $H$ par cette isomorphisme?  
(i) $G$ et $A$ sont-ils décomposables?

**Problème 2.** Les questions (1) et (2) sont indépendantes.

(1) Soit $A$ un anneau, $x \in A$ et $(\alpha, \beta) \in \mathbb{Z}^2$ tel que $\alpha x = \beta x = 0$. Montrer que $(\alpha \wedge \beta)x = 0$ où $\alpha \wedge \beta$ désigne le pgcd de $\alpha$ et $\beta$. Indication: penser à utiliser l'algorithme d'Euclide.

(a) Montrer que $N$ est un idéal de $A$.  
(b) Soit $J$ un idéal de $A$ et $v(J) := \{x \in A, \exists n \in \mathbb{N}^*, x^n \in I\}$. Montrer que $v(J)$ est un idéal de $A$.

**Problème 3.** Soit $\omega := \sqrt{5} + 2i\sqrt{3}$ où $i$ désigne le nombre complexe de module 1 et d'argument $\pi/2$.

(a) Montrer que $\omega$ est algébrique sur $\mathbb{Q}$ et que $\sqrt{5}$ appartient à $\mathbb{Q}(\omega)$. On rappelle que $\mathbb{Q}(\omega)$ désigne l'extension de $\mathbb{Q}$ par $\omega$.  
(b) Déterminer le polynôme minimal de $\omega$ sur $\mathbb{Q}$, la dimension de $\mathbb{Q}(\omega)$ sur $\mathbb{Q}$ et une base de $\mathbb{Q}(\omega)$ sur $\mathbb{Q}$.
le 6 janvier 2010

 Examen, durée : 1h30

> Compléter à l'aide de mots dont on vous donne les premières lettres ainsi qu'un synonyme entre parenthèses. (5pts)

1. Prices r…………… between $50 and $250. (vary)
2. We need to w…………… o…………… the total cost of the project. (calculate, 2 words)
3. The job will take ap…………… three weeks. (roughly, about)
4. I've n…………… finished that book you lent me. (almost)
5. The books were ra…………… arranged on the shelves. (by chance rather than according to a plan)
6. The conference takes place every o…………… year. (second)
7. N…………… and t…………… there are violent cyclones in the Gulf of Mexico. (from time to time)
8. Orders for new ships are rising, after several years of st…………… decline. (regular)
9. She gave a s…………… of lectures at Warwick University last year on contemporary British writers. (sequence, succession)
10. I was amazed at the s…………… of their garden. (dimensions)
11. The boat is ten metres in l…………… (It is ten metres long)
12. After heavy rains, the w…………… of the Amazon can reach 50 kilometres. (It can be 50 kilometres wide)
13. The pa…………… of family life has been changing over recent years. (model)
14. I'm sl…………… upset that she forgot my birthday. (a little)
15. Their m…………… weight was 76.4 kilos. (average)
16. Now that we have a baby, we s…………… get the chance to go to the cinema. (rarely)
17. A recent su…………… found that 58% of people did not know where their heart is. (study, examination of opinions)
18. There is a plan to le…………… the three-year course to four years. (make longer)
19. We are currently flying at a h…………… of 15,000 metres. (altitude)
20. The weather there can be a bit un…………… - one minute it's blue skies and the next minute it's pouring down. (difficult to forecast)
21. The smoke th…………… rapidly. (became thicker)
22. The road n…………… after the bridge. (becomes less wide)
23. The machine makes thousands of me…………… every day. (the process of measuring)
24. You should always e…………… your oil, water and tyres before taking your car on a long trip. (verify)
25. There has been a crime w…………… in the district lately. (a sudden increase in the number of crimes)
QCM « grammaire et structures » Choisir la bonne réponse (4pts)

1. They have lived in Manchester............... they retired.
   a) whenever b) while
   c) ever since d) as

2. He ..................... in Britain for a couple of years, then emigrated to Australia.
   a) has been staying b) stayed
   c) has stayed d) was staying

3. Mr Sack took his new Ferrari for a test spin: « I ..................... 7 miles, when I heard this banging and I saw smoke coming out! ».
   a) was driving b) had driven
   c) will drive d) have driven

4. He said he would come when the strike ..................... over.
   a) was b) would be
   c) should be d) will be

5. This is the worst case our doctor has ..................... seen
   a) still b) never
   c) ever d) already

6. That’s impossible. It couldn’t have happened like that! You ..................... wrong!
   a) should be b) should have been
   c) must be d) ought to have been

7. Our company manufactures a wide ..................... of products.
   a) series b) range
   c) grade d) sequence

8. Mrs Pricklet ..................... for over 5 months now.
   a) is dead b) died
   c) has been dead d) was dead

9. Dr Smith isn’t in tonight because ..................... to the hospital.
   a) he might have gone b) he’s able to go
   c) he should have gone d) he’s had to go

10. They ..................... from her lately.
    a) didn’t hear b) haven’t heard
    c) are not hearing d) don’t hear
11. How long ago ...................... the firm?
   a) have they founded       b) they found
   c) did they found          d) found they

12. Donald ................ sixteens tomorrow.
   a) is being                b) going to be
   c) shall be               d) will be

13. She has been a public figure ever since she .................. her autobiography.
   a) has been writing       b) wrote
   c) had been writing       d) had written

14. Profits rose by 5% last year and ................. this year.
   a) did not decrease       b) are not in decrease
   c) are not decreased      d) have not decreased

15. British weather can never be relied ................... - it's always changing.
   a) in                      b) with
   c) on                     d) after

16. Students account .................... the vast majority of our customers.
   a) in                     b) for
   c) of                     d) with

17. You can always depend ................. Michael in a crisis.
   a) on                      b) of
   c) for                   d) with

18. I prefer teaching methods that actively involve students ................. learning.
   a) with                   b) of
   c) in                   d) for

19. Anyone who lives alone has to look after ...................
   a) himself           b) oneself
   c) self              d) itself

20. ....................... you to be in bed by now?
   a) Mustn't            b) Shouldn't
   c) Oughtn't        d) Needn't

21. They ..................... at least ten minutes ago.
   a) ought to leave       b) might leave
   c) should leave         d) must have left

22. That person has been standing there ................... ages.
   a) since               b) while
   c) for                 d) during

23. “I’m surprised this accident happened.”
    “Yes. He .................. the obstacle.”
   a) should have seen    b) should see
   c) would have seen      d) had to see
24. We are afraid he ........................................... the appointment.
   a) may have forgotten   b) should forget
   c) must forget         d) can have forgotten

➤ Put the verb into the correct form, present perfect (I have done) or past simple (I did). (1,5pts)

Example: I .... didn't play ....... (not / play) golf when I was on holiday last summer.

1. Mr Clark ................................................... (work) in a bank for 15 years.
   Then he gave it up.

2. Molly lives in Dublin. She ........................................ (live) there all her life.

3. Bob and Alice are married. They ........................................ (be) married for 20 years.

4. When we were on holidays, the weather ........................................... (be) awful.

5. My grandfather died 30 years ago. I ........................................... (never / meet) him.

6. I don't know Carol’s husband. I ........................................... (never / meet) him.

➤ Complete the sentences with the verbs in brackets. (1,5pts)

Example: If I could speak Japanese, I (spend) ... would spend ........................................... next year studying in Japan.

1. If I had known Mr Jung was in hospital, I (send) ........................................... him a note and some flowers.

2. It’s too bad that it’s snowing. If it (snow, not) ........................................... we could go for a drive.

3. I was very tired. Otherwise, I (go) ........................................... to the party with you last night.
• Questions about the texts studied in class:

1. What do some animal rights activists do to alert public opinion to the ill-treatment of animals? What do you think of such campaigns? Do you think animals should have rights? Why or why not? Do you think human rights are more important than animal rights? Why or why not? (4pts)

2. Why does the United Nations want to connect a number of African villages to cellular networks? Explain the interests – and also the limitations – of the programme? Do you think high technology can be the solution to the problems of developing countries? Explain. (4pts)
Examen 1ère session, durée : **1h30**

**Compléter les mots dont on vous donne la première lettre ainsi qu'un synonyme entre parenthèses. (4pts)**

1. These scandals will not en................. the organization's image. (improve)
2. The government plan to ra................. taxes. (increase)
3. Dan's actually quite nice, un................. his father. (contrary to)
4. The city lifestyle seems to su................. her. (correspond to, be right for her)
5. The lowering of interest rates will bo................. the economy. (improve)
6. Do you think these two colours m.................? (go together)
7. What an ut................. stupid thing to do! (completely)
8. They did a th................. revision of the manuscript. (detailed and careful)
9. I'm fa................. sure that this is the right address. (quite)
10. This is one of the country's fo................. arts centres. (first, leading)
11. There has been an increase of ro................. 2.25 million. (approximately)
12. Orders for new ships are rising, after several years of st................. decline. (regular)
13. It's an area of ou................. natural beauty. (remarkable)
14. John is very re................. - if he says he'll do something, he'll do it. (dependable, you can trust him)
15. Al................. all the passengers on the ferry were French. (nearly)
16. I'm sl................. upset that she forgot my birthday. (a little)
17. The village has remained ba................. unchanged for over 300 years. (fundamentally)
18. Do you play any other sports be................. football and basketball? (in addition to, apart from)
19. I didn't ac................. see her - I just heard her voice. (in fact, really)
20. You eat a massive plate of food for lunch, wh................. I have just a sandwich. (compared with the fact that; but)
21. I knew a lot about the subject already, but her talk was interesting ne................. . (however)
22. Diets that are high in saturated fat and cholesterol tend to clog up our arteries, th................. reducing the blood flow to our hearts and brains. (thus)
23. He was in tears and ob................. very upset. (clearly)
24. As ................. , I only read detective novels. (generally – three words)
Compléter à l'aide du pronom relatif qui convient :
(2pts)

1. She heard a noise and couldn't say ................ it was.

2. Bogart's face suggested a man ................ had experienced life and had been hardened by it.

3. It's the only thing ................ would soften him.

4. It is the concert ............... you should have gone to.
   a) what  b) ∅  c) whose  d) whom

5. She called the driver, behind ............... I was sitting.
   a) ∅  b) who  c) which  d) whom

6. She said nothing to those ............... had not listened.
   a) which  b) what  c) ∅  d) who

Reported speech. Complete the sentences by reporting the speaker's words. (2pts)

Examples: Bob said: “I will help you.”
Bob said ….. he would help me …..

“Do you need a pen?” Annie asked.
Annie asked me … if I needed a pen …..

1. Jennifer asked: “What do you want?”
   Jennifer asked me

2. Sid asked: “Are you hungry?”
   Sid wanted to know

3. “I’m going to move to Ohio,” said Bruce
   Bruce informed me

   Connie asked me

5. Dick asked: “What are you talking about?”
   Dick asked me

6. Barbara said: “I have to go downtown.”
   Barbara said.
7. My mother asked: "Why are you tired?"
   My mother wondered ............................................................... 
8. "Why is the sky blue?" my young daughter often asks.
   My young daughter often asks me ..........................................

➢ Mettre l'article the, a, an, lorsque cela est nécessaire.
   Si vous pensez qu'il ne faut rien ajouter marquez Ø. (2pts)

............ oldest science

............ science of ............ astronomy is ............ ancient one. Since earliest times, ............ man has wondered about ............ twinkling lights in ............ sky – why some of them moved while others remained stationary, why ............ moon appeared to grow and shrink in size each month, and why ............ sun seemed to disappear occasionally, swallowed by ............ blackened disk with ............ shimmering halo in ............ eclipse.

Ancient civilisations began to record celestial events in ............ calendars, and began formulating ideas about how ............ universe was structured. One noted Greek astronomer, Hipparchus, mapped ............ constellations and estimated ............ brightness of the stars; another, Ptolemy, devised ............ earth-centred view of ............ universe, where everything, including ............ sun, planets and stars revolved around ............ earth.

➢ Put the verb into the correct form, -ing or to+infinitive. (2pts)

Examples: If you want to pass your exams, you need .... to study...... (study) more. I’m sorry I broke the glass. I couldn’t help .... dropping...... (drop) it.

1. Would you like......................... (come) to a party?
2. I can’t stand people......................... (tell) me what to do when I’m driving.
3. I very much enjoy......................... (listen) to classical music.
4. Those shirts need ......................... (iron), but you don’t need......................... (do) it now.
5. Ann offered ......................... (look) after our children while we were out.
6. Sue is looking forward to ......................... (go) on holiday.
7. She ran five miles without ......................... (stop).
Répondre en anglais aux questions suivantes portant sur les textes étudiés en classe.

1. What is the « lighthouse »? What are its features and its advantages? Would you like to live in a house like this? Describe the house where you would like to live. (4pts)

2. Give examples of bullying at school and explain the problems posed by such a practice. How do you think bullying could be discouraged? Explain. (4pts)